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A Refractive-Diffractive element Design for resonant scanner angular correction: The symmetric beam retardation approach

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Abstract

We propose a new approach for designing diffractive optical corrective elements with zooming capability to convert nonlinear sinusoidal scanning into linear scanning. Such devices will be useful for linearizing the angular scan of a resonant mirror scanner. The design methodology is to create a graded index of refraction device as the reference design, with its index of refraction parameters based on beam retardation through propagation in an inhomogeneous media. In a new approach, we make explicit use of the symmetry of the optical problem to mathematically derive the form of the refractive gradient. The diffractive element is designed utilizing a binarizing algorithm of the accumulated phase from transmission through the refractive element.

1. Introduction

Diffraction optical elements have found considerable use in many areas of optics. One important area of application is as a corrector element in optical systems to modify wavefronts to remove distortions or aberrations. For example, a diffraction element has been designed to provide compensation for thermally induced aberrations in a solid-state laser¹. In applications such as these, diffraction technology provides a planar, lithographically fabricable solution to replace difficult to realize 3-D conventional refractive optics². Diffraction elements can also be utilized in important applications such as beam steering³ and optical free space interconnects⁴. In the present work we seek to find a diffraction solution to the important practical problem of distortion in fast optical scanners.

High speed optical scanning of laser beams has found numerous applications in science, engineering and medicine, from the ubiquitous laser printer to geophysical remote sensing and confocal microscopy. An optomechanical device known as a resonant scanner is often used for scanning lasers. Resonant scanners, which are oscillating mirrors excited to operate at their natural resonant frequency, provide a nonlinear sinusoidal scan, and at the limits of the scan, where the sine function changes more slowly near its peak, the scan field is significantly compressed. Despite this deficiency, resonant scanners have many attractive attributes and are widely used. In imaging applications, the scanned image can be electronically corrected during or after acquisition to produce an image equivalent to which would be produced by an ideal linear scanner,^{5,6} whereas other applications require that the scan itself must be made linear. An optical means of correction would be highly desirable, since it means that the wide variety of

commercially available resonant scanners could be utilized without any modification to the scanner itself.

In previous work, we have reported on the derivation of a diffractive element and its associated refractive element for the very same purpose. The derivation of the graded index of refraction of the refractive active element in our prior work was based on beam propagation in an inhomogeneous media⁷ and on the beam retardation approach⁸. The diffractive element is simply a binary phase or amplitude version of the refractive element. In the present work, we describe a new design based on the beam retardation approach (phase lag), which explicitly takes into account the symmetry of the optical problem. As with our earlier work⁹, which we will refer to as the ‘asymmetric solution’ the present design does take the beam diameter into consideration.

Comparison of the first order approximation solutions of the beam propagation and the beam retardation approaches, suggests that for a quasi broad beam, the solution from the beam propagation approach should collapse to the solution from the beam retardation approach at this level of approximation. As we shall see later, the differential equations we develop only contain a linear dependence on the ratio of the element size to the beam diameter ΔD . These considerations suggest that this new variant of the beam retardation approach will work well with narrow to moderately broad beams. The addition of the higher order terms will naturally improve the accuracy of our computation.

2. Device Theory

Our theory for developing the index of refraction of the corrective element for converting sinusoidal to linear scanning consist as follow: In Fig. 1, let us assume that we have a corrective optical element which is located at a distance equal to Z_2 from a sinusoidal

scanner. The reflected beam out of the scanner is incident upon the corrective element in the y axis at h_2 and at an angle θ_2 which is proportional to $\sin(\omega t)$, where ω is the scanning angular frequency. The corrective element deflects the output beam, which propagates at angle θ_3 , and meets the target plane at point b. This point b is located at the target plane at the highest of h from the axis and at a distance of $Z_1 - Z_2$ from the corrective element (or Z_1 from the scanner). Point b corresponds to the point where the beam should arrive if the sinusoidal scanner is replaced by a linear scanner. The corresponding virtual scan angle is θ_1 , as shown in the diagram.

For developing the design of the required corrective element, we use the beam retardation approach. In this method, we consider that we have a corrective element with a graded index of refraction. When a beam propagates through the graded index of refraction material, it causes one side of the beam to phase lag behind the other. This phase lag causes the beam wavefront to bend. We derived the exact formula for this refractive element; however, since it is hard to fabricate a refractive element with that level of complexity of the gradient in the index of refraction, or its corresponding diffractive element, we use a binarization scheme to modify the results to produce a diffractive element that is easier to fabricate.

Before we derive the formula for the corrective element, it is essential that we derive the output angle θ_3 from this optical element as a function of θ_1 , the desired linear scan angle. That is, we want a closed form mathematical expression of the functionality of the element.

From simple geometrical considerations in Fig. 1 it is possible to show that the linear scan angle θ_1 and the input scan angle θ_2 to the corrective element are given by,

$$h = Z_1 \tan \theta_1, \quad (1)$$

and,

$$h_2 = Z_2 \tan \theta_2. \quad (2)$$

From Fig. 1 and equations 1 and 2 it is possible to show that the output deflection angle θ_3 is given by,

$$\tan \theta_3 = \frac{h - h_2}{Z_1 - Z_2} = \frac{Z_1 \tan \theta_1 - Z_2 \tan \theta_2}{Z_1 - Z_2}, \quad (3A)$$

And in the paraxial approximation,

$$\theta_3 = \frac{h - h_2}{Z_1 - Z_2} = \frac{Z_1 \theta_1 - Z_2 \theta_2}{Z_1 - Z_2}. \quad (3B)$$

For sinusoidal scanning it possible to express the scan angle θ_2 in the form,

$$\theta_2 = \theta_{20} \sin \omega t = \frac{y}{Z_2}, \quad (4A)$$

where θ_{20} is the maximum scanning amplitude and ω is angular frequency for scanning.

For calculating the graded index of refraction of the refractive reference element, as a function of the y coordinate, it is necessary to express the input and the output angles in terms of the y coordinate instead of the time. Equation (4A) can be written in the inverse function form as:

$$t = \frac{1}{\omega} \sin^{-1} \frac{y}{Z_2 \theta_{20}} \quad (4B)$$

On the other hand, if we assume that the sinusoidal scanner is replaced by a linear scanner, then it is possible to write the equation of motion as

$$\theta_1 = gt, \quad (5)$$

where g is a constant. Substituting the time value of equation 4B in equation 5 yields

$$\theta_1 = \frac{g}{\omega} \sin^{-1} \left(\frac{y}{Z_2 \theta_{20}} \right). \quad (6)$$

Substituting equations 6 and 4A in equation 3, yields the output of the diffracted beam as a function of y , given by the following equation:

$$\tan \theta_3 \approx \theta_3 = \frac{Z_1 \frac{g}{\omega} \sin^{-1} \left(\frac{y}{Z_2 \theta_{20}} \right) - Z_2 \frac{y}{Z_2}}{(Z_1 - Z_2)} = \frac{g Z_1 \sin^{-1} \left(\frac{y}{Z_2 \theta_{20}} \right) - y \omega}{(Z_1 - Z_2) \omega}, \quad (7)$$

which mathematically represents the action of our corrective element. We now proceed to the detailed design of the element.

2.1 Beam retardation approach

Let us assume as shown in figure 2 that there is a beam of diameter ΔD propagating in a graded index of refraction medium of thickness l . The medium has a graded index of refraction in the y direction. As mentioned earlier, during propagation, one side of the beam phase lags behind the other side, which in turn makes the beam wave front bend.

The phase lags of the upper and lower sides of the beam respectively are:

$$\theta_4 = \frac{2\pi(n_0 + \Delta n)l}{\lambda}, \quad (8)$$

$$\text{and } \theta_5 = \frac{2\pi(n_0)l}{\lambda}. \quad (9)$$

Therefore the beam wavefront bends by an angle of

$$\Delta \theta_{45} = \theta_4 - \theta_5 = \frac{2\pi(\Delta n)l}{\lambda}. \quad (10)$$

The deviation as a result of the phase lag can be written as,

$$\Delta \theta_{45} \cong \frac{2\pi}{\lambda} \Delta n (\Delta D) \ell. \quad (11)$$

For conversion of sinusoidal scanning to linear scanning, the beam deflection due to propagation through the refractive element should be equal to the difference in the propagation angle between sinusoidal and linear scanning, e.g.

$$\Delta\theta_{45} = \Delta\theta_{32} = \theta_3 - \theta_2. \quad (12)$$

Equalizing equations 11 and 12, and using equation 3B for the value of θ_3 yields

$$\frac{2\pi}{\lambda} \Delta n(\Delta D)\ell = \theta_3 - \theta_2 = \frac{Z_1\theta_1 - Z_2\theta_2}{Z_1 - Z_2} - \theta_2. \quad (13)$$

Substituting the values of θ_1, θ_2 from equations 4A and 6 into equation (13) yields

$$\frac{2\pi}{\lambda} \Delta n(\Delta D)\ell = \frac{Ag}{\omega} \sin^{-1}\left(\frac{y}{z_2\theta_{20}}\right) - A\frac{y}{z_2}. \quad (14)$$

Since a resonant scanner deflects symmetrically about its central rest position, the corrective optical element that corrects the deflected beam for nonlinear scanner deflection must have a graded refractive index that is symmetrical with respect to the central or zero position.

In order to solve the differential equation (14), we need to replace Δn by its Taylor series expansion. For a symmetric function around the origin, Δn can be reduced further. The Taylor series expansion of the index of refraction at the origin is,

$$n(x) = n_0 + a_1 \frac{\partial n}{\partial x} x + a_2 \frac{\partial^2 n}{\partial x^2} x^2 + a_3 \frac{\partial^3 n}{\partial x^3} x^3 + a_4 \frac{\partial^4 n}{\partial x^4} x^4 + \dots, \quad (15)$$

where n_0 is the offset (the refractive index at the center), and a_1, a_2, a_3 etc. are given

by $a_1 = 1, a_2 = \frac{1}{2!}, a_3 = \frac{1}{3!}, a_4 = \frac{1}{4!}$ and so forth.

Since the diffractive element index of refraction $n(x)$ should be symmetric around the point of symmetry, the terms containing the odd powers in x should be eliminated, thereby reducing the Taylor series expression for the index of refraction to:

$$n(x) = n_0 + a_2 \frac{\partial^2 n}{\partial x^2} x^2 + a_4 \frac{\partial^4 n}{\partial x^4} x^4 + \dots \quad (16)$$

The graded index of refraction $\Delta n(x)$ can now be derived, and therefore,

$$\Delta n(x) = 2a_2 x \frac{\partial^2 n}{\partial x^2} \Delta x + 4a_4 x^3 \frac{\partial^4 n}{\partial x^4} \Delta x + 6a_6 x^5 \frac{\partial^6 n}{\partial x^6} \Delta x + \dots \quad (17)$$

Substituting expression (17) for $\Delta n(x)$ in equation (14), and remembering $z_2 \theta_{20} = y_0$, yields,

$$\frac{2\pi}{\lambda} \left[2a_2 y \frac{\partial^2 n}{\partial y^2} + 4a_4 y^3 \frac{\partial^4 n}{\partial y^4} + 6a_6 y^5 \frac{\partial^6 n}{\partial y^6} \right] (\Delta D) l = \frac{Ag}{\omega} \sin^{-1} \left(\frac{y}{y_0} \right) - A \frac{y}{Z_2}, \quad (18)$$

where $a_2 = \frac{1}{2!}$, $a_4 = \frac{1}{4!}$, $a_6 = \frac{1}{6!}$.

It was not possible to directly solve differential equation (18). To overcome the problem of not having a direct solution, the arcsine on the right hand side of the equation was replaced by its polynomial expansion to the seventh order. Figure 3 shows $\sin^{-1}(x)$ and its polynomial expansion to the seventh and the ninth orders. Using this approach, Eqn. (18) was solved for both the second and the fourth order Taylor expansions of the index of refraction.

The solution for the second order expansion is,

$$n(y) = \frac{5B_0 y_0}{6272} \left(\frac{y}{y_0} \right)^8 + \frac{B_0 y_0}{400} \left(\frac{y}{y_0} \right)^6 + \frac{B_0 y_0}{72} \left(\frac{y}{y_0} \right)^4 + \frac{(B_0 - B_1 y_0) y_0}{2} \left(\frac{y}{y_0} \right)^2 + C_1, \quad (19)$$

where $B_0 = \frac{Ag\lambda}{2\pi(\Delta D)\omega l}$, and $B_1 = \frac{A\lambda}{2\pi(\Delta D)Z_2 l}$.

Similarly the solution for the fourth order expansion is,

$$n(y) = \frac{5B_0 y_0}{225792} \left(\frac{y}{y_0} \right)^8 + \frac{B_0 y_0}{7200} \left(\frac{y}{y_0} \right)^6 + \frac{B_0 y_0}{576} \left(\frac{y}{y_0} \right)^4 + \frac{y^2 (B_0 - 6B_1 y_0) y_0}{12} \left(\frac{y}{y_0} \right)^2 + C_1. \quad (20)$$

The details on the derivation of the above equation can be found in the appendix A.

The ratio g/ω in both equations 14 and 18, which is also expressed within the parameter B_0 of equations 19 and 20, has significance in the design of the diffractive element. The ratio of g/ω determines the zooming capability or the magnification of the corrective element either to be equal, less or larger than the original uncorrected field of view. Looking at Fig. 4, which compares the functions of motion for both linear and nonlinear scanning, reveals the following: in order for the corrective element to have no effect on the dimensions of the field of view (FOV), it is necessary for the amplitudes of both the sinusoidal and triangular motions to be equal. This is satisfied when $g = 2\omega/\pi$ and $g/\omega = 2/\pi$. On the other hand, in order for the diffractive element to maintain the linear portion of the sinusoidal scanning near the nodal line, the tangent of the triangular function should be equal to tangent of the sinusoidal function; e.g. $g = \omega$. Also, an examination of equations 19 and 20 shows that increasing the ratio $y_0/\Delta D$ (remembering that B_0 and B_1 contain $1/\Delta D$) will require an increase in the index of refraction n for the corrective element.

2.2 Binarization Algorithm

Fabrication of a corrective element with a complex graded index profile based on equations 19 and 20 is not easy. The gradient index refractive element is therefore best

used as a reference design. Hence, we propose to build a binary diffractive element for image corrections that has the following transmissivity:

For binary amplitude encoding:

$$T = \begin{cases} 2(m-1)\frac{2\pi}{b_l} < \frac{2\pi}{\lambda}nl < 2m\frac{2\pi}{b_l} = 0 \\ (2m-1)\frac{2\pi}{b_l} < \frac{2\pi}{\lambda}nl < (2m+1)\frac{2\pi}{b_l} = 1 \end{cases} \quad (21)$$

and for binary phase encoding:

$$T = \begin{cases} 2(m-1)\frac{2\pi}{b_l} < \frac{2\pi}{\lambda}nl < 2m\frac{2\pi}{b_l} = -1 \\ (2m-1)\frac{2\pi}{b_l} < \frac{2\pi}{\lambda}nl < (2m+1)\frac{2\pi}{b_l} = 1 \end{cases} \quad (22)$$

where b_l is the number of binarization levels; if $b_l=2$, the encoding is binary, $b_l=3$, the encoding is ternary, if $b_l=4$, the encoding is quaternary and so on.

Selection of the offset value [the constant C_1 in equations (19) and (20)] is extremely important for the design of the binary amplitude version of the diffractive element. This is not so critical in the binary phase version of the diffractive element and for the refractive element. In the design of the binary amplitude version of the diffractive element one should be sure that the center of the diffractive element is transmissive.

In order to have a transmissive diffractive element at the origin, the phase delay through the transmission at the origin of the refractive element should satisfy the following condition for its value:

$$\frac{2\pi n(0)l}{\lambda} = (2m+1)\frac{2\pi}{b_l}, \quad (23)$$

where $n(0)$ is the index of refraction at the origin (e.g. at $y=0$), and in equations (19) and (20) is equal to the constant of integration " C_1 ". Rearranging (23) we have,

$$m = \frac{1}{2} \left[\frac{n(0)lb_l}{\lambda} - 1 \right] \quad (24)$$

Setting $n(0) \approx 1$, is a reasonable selection of the graded index of refraction at the origin, in order to estimate m . We thus obtain,

$$m = \frac{1}{2} \left[\frac{lb_l}{\lambda} - 1 \right]. \quad (25)$$

For the generation of the diffractive element we selected:

$$m = \text{round} \left[\frac{1}{2} \left(\frac{lb_l}{\lambda} - 1 \right) \right], \quad (26)$$

and the constant of integration from equations 19 and 20 is thus,

$$C_1 = \frac{\left[2 \text{round} \left[\frac{1}{2} \left(\frac{lb_l}{\lambda} - 1 \right) \right] + 1 \right] \lambda}{lb_l}. \quad (27)$$

2.3 Simulations

We utilize commercial software (MATLAB, MathWorks, Natick, MA) to compute, process, and display the results obtained from our implementation of our corrective element design. Figure 5 depicts drawings showing the diffractive elements generated from the second order approximation of our algorithm by binarization, for a constant aperture of 1 cm for different beam diameters (a) $\Delta D = 0.01$ cm, (b) $\Delta D = 0.005$ cm and (c) $\Delta D = 0.002$ cm. From these drawings, we can see that the binary diffractive element consists of a chirped grating whose frequency increases as it goes to the lateral edges of the diffractive element. An increase in the grating frequency is associated with an increase in the graded index of refraction of the corresponding refractive element. As expected intuitively, smaller beam diameters require smaller grating spacing for the

diffractive element to function properly, and a more exact reason may be found by examining Figs. 6 and 8. Figure 6 shows plots of the graded index of refraction for the second order approximation using equation 17. In all of these plots it was assumed that the refractive element thickness $l = 1\text{mm}$. (a) The diffractive element aperture size was kept constant ($y_0 = 1\text{ cm}$), while three different values of the beam diameter ΔD were considered; (b) the beam diameter ΔD of the diffractive element was kept constant at 0.01 cm while three different values of the diffractive element aperture y_0 , 0.5 , 1 and 2cm , were considered. It is evident from the plots in 6(a), that beams with smaller diameters require a diffractive element with a higher graded index of refraction in order to achieve the same beam propagation correction, which is reflected in the higher grating frequencies we observe in Fig. 5. The plots in 6(b) reveal that larger corrective element apertures also require a larger graded index of refraction,

Figure 7 shows the same plots in the same order as in figure 5, but using the graded index of refraction for fourth order approximations derived in equation 20. The drawings are for a constant aperture of 1 cm , for different beam diameters; (a) $\Delta D = 0.001\text{ cm}$, (b) $\Delta D = 0.0005\text{ cm}$ and (c) $\Delta D = 0.0002\text{ cm}$. Similarly Fig. 8 shows plots similar to those in Fig. 6, but derived from the more accurate fourth order approximations instead of the second order expressions utilized for Fig. 6. For Fig. 8(a), the aperture size was kept constant, while the beam diameter took on three different values of 0.02 mm (0.002 cm), 0.05 mm (0.005 cm) and 0.1 mm (0.01 cm). Again, we observe that beams with smaller diameters are associated with higher graded indices of refraction. This can be explained by considering that a beam with a larger diameter requires less graded index of refraction in order to achieve the same phase lag between the two sides of the beam. For a beam with

an infinitesimal diameter, the graded index of refraction needs to be extremely large to accumulate a sufficient beam lag between the two sides of the beam. In Fig. 8(b), two different values of the aperture size y_0 , 1 and 0.5 cm were utilized, whereas the beam diameter was kept constant at 0.01 mm (0.001 cm). The larger aperture is associated with a small increase in the graded index of refraction.

2.4 Results and Discussion

We have presented a formula for the graded index of refraction, in equation 17, and two solutions of different degrees of approximation in eqns. 19 and 20. In each case, the value of the refractive index exhibits a linear dependence on the ratio of the dimension of the diffractive element and the value of the beam diameter ΔD , i.e. the use of a refractive element or its associated diffractive element fabricated in accordance with the appropriate value of the beam diameter is thus desirable, while using a diffractive element intended for correcting a different diameter beam is less deleterious than in most other design methods.

When we examine a section of the diffractive elements shown in Fig. 5 or Fig. 7 we see that any portion of the diffractive element consists of a chirped grating. Thus we expect a problem with beam fanning. If the grating consists of equally spaced lines, then the diffraction pattern from such a grating will be distinguished by diffraction order numbers. However, because this grating has no constant spacing, the left side of beam will diffract more than the right hand side of the beam and this causes beam fanning in addition to producing regular diffraction orders around the average line spacing. This can be minimized by careful attention to system requirements and designing for the appropriate beam diameter.

3. Conclusions

In conclusion, we have proposed a refractive corrective element with zooming capability for converting nonlinear sinusoidal scanning into linear scanning. We derived expressions for the graded index of refraction for the refractive element based on the beam retardation approach through propagation in inhomogeneous media. In a new approach, we make explicit use of the symmetry of the optical problem to mathematically derive the form of the refractive gradient. An algorithm for converting the graded index of refraction design to the binary diffractive version is also presented. In our theory, we found that the index of refraction increases as we moved to the lateral edges of the corrective element, behaving like a diverging lens. In the binary version of this element, this has been expressed by having a chirped grating with an increasing frequency as one moves from the center to the sides. We also find from the theoretical derivation, and as expressed in the computer simulations, that the graded index of refraction increases with the requirement of an increase in the aperture of the device and a decrease in the beam diameter. In the binary diffractive element, this has been expressed by a concomitant increase in the frequency of the chirped grating. Thus for best results the diffractive element should be designed for the actual beam diameter in use. This would also minimize the undesired beam fanning inherent in the use of a chirped grating, as discussed earlier. The performance of the diffractive device should be adequate for many applications and replicated devices could be fabricated at low cost.

Appendix A

The Taylor series expansion of the index of refraction n of the refractive element, around the point of symmetry ($x=0$) can be written as

$$n(x) = n_0 + a_1 \frac{\partial n}{\partial x} x + a_2 \frac{\partial^2 n}{\partial x^2} x^2 + a_3 \frac{\partial^3 n}{\partial x^3} x^3 + a_4 \frac{\partial^4 n}{\partial x^4} x^4 + \dots \quad (\text{A1})$$

where n_0 is the index of refraction at the origin (i.e. the point of symmetry), and a_1, a_2, a_3, a_4 etc. are the Taylor expansion coefficients given by

$$a_1 = 1, a_2 = \frac{1}{2!}, a_3 = \frac{1}{3!}, a_4 = \frac{1}{4!}, a_5 = \frac{1}{5!}, a_6 = \frac{1}{6!}.$$

n is a symmetric function around the origin. This causes all the odd powers of x to be cancelled in the expansion and we just keep the even powers.

$$n(x) = n_0 + a_2 \frac{\partial^2 n}{\partial x^2} x^2 + a_4 \frac{\partial^4 n}{\partial x^4} x^4 + \dots \quad (\text{A2})$$

Similarly, the Taylor series expansion around the coordinate $x+\Delta x$ is

$$n(x + \Delta x) = n_0 + a_2 \frac{\partial^2 n}{\partial x^2} (x + \Delta x)^2 + a_4 \frac{\partial^4 n}{\partial x^4} (x + \Delta x)^4 + \dots \quad (\text{A3})$$

This leads to the variation in the index of refraction to be,

$$\Delta n(x) = n(x + \Delta x) - n(x); \quad (\text{A4})$$

$$\text{since, } \begin{cases} (x + \Delta x)^2 - x^2 = \Delta(x^2) \cong 2x\Delta x \\ (x + \Delta x)^4 - x^4 = \Delta(x^4) \cong 4x^3\Delta x \\ (x + \Delta x)^6 - x^6 = \Delta(x^6) \cong 6x^5\Delta x \end{cases}$$

we have,

$$\Delta n = 2a_2 x \frac{\partial^2 n}{\partial x^2} \Delta x + 4a_4 x^3 \frac{\partial^4 n}{\partial x^4} \Delta x + 6a_6 x^5 \frac{\partial^6 n}{\partial x^6} \Delta x, \quad (\text{A5})$$

which is Eqn. (17) in the body of the paper.

We consider the beam retardation for a beam of width ΔD traversing a graded index material whose refractive index is described by $n(y)$. The value of Δn in Eqn. (A5) can now be substituted in the expression for beam retardation in our design equation (18), so we have:

$$\frac{2\pi}{\lambda}(\Delta n)(\Delta D)l = \frac{Ag}{\omega} \sin^{-1}\left(\frac{y}{y_0}\right) - A \frac{y}{Z_2}, \quad (\text{A6})$$

or, after the substitution and dividing both sides by l , we have:

$$\frac{2\pi}{\lambda} \left[2a_2 y \frac{\partial^2 n}{\partial y^2} + 4a_4 y^3 \frac{\partial^4 n}{\partial y^4} + 6a_6 y^5 \frac{\partial^6 n}{\partial y^6} \right] (\Delta D) = \frac{Ag}{\omega l} \sin^{-1}\left(\frac{y}{y_0}\right) - A \frac{y}{Z_2 l} \quad (\text{A7})$$

Multiplying both sides by y and dividing both sides of the equation by $2\pi(\Delta D)/\lambda$

we obtain:

$$2a_2 y^2 \frac{\partial^2 n}{\partial y^2} + 4a_4 y^4 \frac{\partial^4 n}{\partial y^4} + 6a_6 y^6 \frac{\partial^6 n}{\partial y^6} = \frac{Agy}{2\pi\omega l} \sin^{-1}\left(\frac{y}{y_0}\right) - \frac{A}{(\Delta D)2\pi} \cdot \frac{y^2}{Z_2 l}. \quad (\text{A8})$$

We now substitute $a_0 = 2a_2$, $a_1 = 4a_4$ and $a_2 = 6a_6$ and get:

$$a_0 y^2 \frac{\partial^2 n}{\partial y^2} + a_1 y^4 \frac{\partial^4 n}{\partial y^4} + a_2 y^6 \frac{\partial^6 n}{\partial y^6} = \frac{Agy}{2\pi\omega l} \sin^{-1}\left(\frac{y}{y_0}\right) - \frac{A}{(\Delta D)2\pi} \cdot \frac{y^2}{Z_2 l} = F(y). \quad (\text{A9})$$

This is a homogeneous differential equation of the form

$$a_0 y^p \frac{d^p n}{dy^p} + a_1 y^{p-1} \frac{d^{p-1} n}{dy^{p-1}} - \dots - a_{p-1} y \frac{dn}{dy} + a_p n = F(y). \quad (\text{A10})$$

One way to solve this differential equation is to use the Cauchy-Euler approach,

through variable substitution, where we substitute $y = e^t$.

Substituting $y = e^t \rightarrow t = \ln y$, for $y > 0$ in the equation yields

$$A_0 \frac{d^6 n}{dt^6} + A_1 \frac{d^5 n}{dt^5} + A_2 \frac{d^4 n}{dt^4} + A_3 \frac{d^3 n}{dt^3} + A_4 \frac{d^2 n}{dt^2} + A_5 \frac{dn}{dt} = G(t), \quad (\text{A11})$$

where,

$$G(t) = B_0 e^t \sin^{-1}\left(\frac{e^t}{y_0}\right) - B_1 e^{2t}, \quad (\text{A12})$$

and

$$A_0 = \frac{1}{120}, A_1 = -\frac{1}{8}, A_2 = -\frac{83}{120}, A_3 = -\frac{49}{40}, A_4 = \frac{29}{20}, A_5 = -3, \text{ and}$$

$$B_0 = \frac{Ag\lambda}{2\pi(\Delta D)\omega l}, B_1 = \frac{A\lambda}{2\pi(\Delta D)Z_2 l}.$$

This equation is not trivial to solve. We expanded the arcsine function in a series to the seventh order:

$$\sin^{-1}\left(\frac{e^t}{y_0}\right) = \frac{e^t}{y_0} + \frac{1}{6} \frac{e^{3t}}{y_0^3} + \frac{3}{40} \frac{e^{5t}}{y_0^5} + \frac{15}{336} \frac{e^{7t}}{y_0^7}. \quad (\text{A13})$$

This yielded the expression,

$$A_0 \frac{d^6 n}{dt^6} + A_1 \frac{d^5 n}{dt^5} + A_2 \frac{d^4 n}{dt^4} + A_3 \frac{d^3 n}{dt^3} + A_4 \frac{d^2 n}{dt^2} + A_5 \frac{dn}{dt}$$

$$= B_0 \left[\frac{e^{2t}}{y_0} + \frac{1}{6} \frac{e^{4t}}{y_0^3} + \frac{3}{40} \frac{e^{6t}}{y_0^5} + \frac{15}{336} \frac{e^{8t}}{y_0^7} \right] - B_1 e^{2t} \quad (\text{A14})$$

Even with this simplification it is not possible to find an exact solution to the problem. Therefore we considered simplified solutions for second order and fourth order expansions of the equation (A9).

(a) Second order solution

Through ignoring the 4th and 6th order differential terms in Eqn.(A9), by following the same steps as we did in obtaining equation (A14) which included these terms to the 6th order in its derivation, we found the following relationship:

$$\frac{d^2 n}{dt^2} - \frac{dn}{dt} = B_0 e^t \sin^{-1}\left(\frac{e^t}{y_0}\right) - B_1 e^{2t} \quad (\text{A15})$$

The solution of the above differential equation is:

$$n(t) = \frac{5B_0 e^{8t}}{6272 y_0^7} + \frac{B_0 e^{6t}}{400 y_0^5} + \frac{B_0 e^{4t}}{72 y_0^3} + \frac{e^{2t}(B_0 - B_1 y_0)}{2 y_0} + C_1 + e^t C_2.$$

Substituting $y = e^t$ yields: (A16)

$$n(y) = \frac{5B_0 y^8}{6272 y_0^7} + \frac{B_0 y^6}{400 y_0^5} + \frac{B_0 y^4}{72 y_0^3} + \frac{y^2(B_0 - B_1 y_0)}{2 y_0} + C_1 + y C_2 \quad (A17)$$

Our assumption since the beginning was that n should be symmetric. Therefore C_2 should be zero, so that we may have a symmetric function. Through multiplication of both the numerator and the denominator of the terms in Eqn. (A17) by y_0 , assuming $C_2 = 0$, and rearranging, it is possible to obtain:

$$n(y) = \frac{5B_0 y_0}{6272} \left(\frac{y}{y_0}\right)^8 + \frac{B_0 y_0}{400} \left(\frac{y}{y_0}\right)^6 + \frac{B_0 y_0}{72} \left(\frac{y}{y_0}\right)^4 + \frac{(B_0 - B_1 y_0) y_0}{2} \left(\frac{y}{y_0}\right)^2 + C_1, \quad (A18)$$

which is the 2nd order solution, shown in equation (19) in the main text.

(b) Fourth order solution

Considering terms to fourth order on the left side of Eqn. (A9), the equation reduces to:

$$a_0 y^2 \frac{\partial^2 n}{\partial y^2} + a_1 y^4 \frac{\partial^4 n}{\partial y^4} = F(y) \quad (A19)$$

Following the same approach as before, the 4th order equation can be written as:

$$\frac{1}{6} \frac{d^4 n}{dt^4} - \frac{d^3 n}{dt^3} + \frac{17}{6} \frac{d^2 n}{dt^2} - 2 \frac{dn}{dt} = B_0 e^t \sin^{-1} \left(\frac{e^t}{y_0} \right) - B_1 e^{2t}, \quad (A20)$$

which has the following solution:

$$\begin{aligned}
n(y) = & \frac{5B_0y^8}{225792y_0^7} + \frac{B_0y^6}{7200y_0^5} + \frac{B_0y^4}{576y_0^3} + \frac{y^2(B_0 - 6B_1y_0)}{12y_0} + C_1 + yC_2 \\
& + y^{5/2} \left(C_3 \cos \frac{\sqrt{23}}{2} \ln y + C_4 \sin \frac{\sqrt{23}}{2} \ln y \right).
\end{aligned} \tag{A21}$$

For n to be an even function,

$C_2 = 0$, $C_3 = 0$, $C_4 = 0$, so we have

$$n(y) = \frac{5B_0y_0}{225792} \left(\frac{y}{y_0} \right)^8 + \frac{B_0y_0}{7200} \left(\frac{y}{y_0} \right)^6 + \frac{B_0y_0}{576} \left(\frac{y}{y_0} \right)^4 + \frac{y^2(B_0 - 6B_1y_0)y_0}{12} \left(\frac{y}{y_0} \right)^2 + C_1, \tag{A22}$$

which corresponds to equation (20) in the main text.

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Figure Captions:

Fig.1. Ray Diagram of Resonant Scanner with Refractive/Diffractive Corrective Element.

Fig 2. Schematic diagram illustrates wavefront bending due to phase lag between the two sides of the beam as the beam propagates in a graded index of refraction material.

Fig 3. Plots of $\sin^{-1}x$ and its polynomial expansion to the seventh and the ninth orders. The solid line is the plot for $\sin^{-1}(x)$, the dotted line the polynomial expansion for the seventh order. The dashed line is the polynomial expansion to the ninth order

Fig. 4. Linearized Scanning Geometries with FOV, greater than, equal to or less than that for the sinusoidal scanner.

Fig. 5. Diffractive element for the second order solution with (a) $\frac{y_0}{\Delta D} = 100$, $y_0=1\text{cm}$ and

$\Delta D=0.01\text{ cm}$, b) $\frac{y_0}{\Delta D} = 200$, $y_0=1\text{cm}$ $\Delta D=0.005\text{ cm}$, (c) $\frac{y_0}{\Delta D} = 500$, $y_0=1\text{cm}$ $\Delta D=0.002$

cm, as generated in MATLAB. The calculated grating spacing is a smooth chirp. The slight banding seen in the figures is an artifact of the way MATLAB displays these patterns.

Fig. 6. Plots of the graded index of refraction as a function of the normalized distance from the optical axis for the second order solutions. (a) For constant y_0 and varying beam diameters ΔD (b) For varying y_0 and constant beam diameter ΔD .

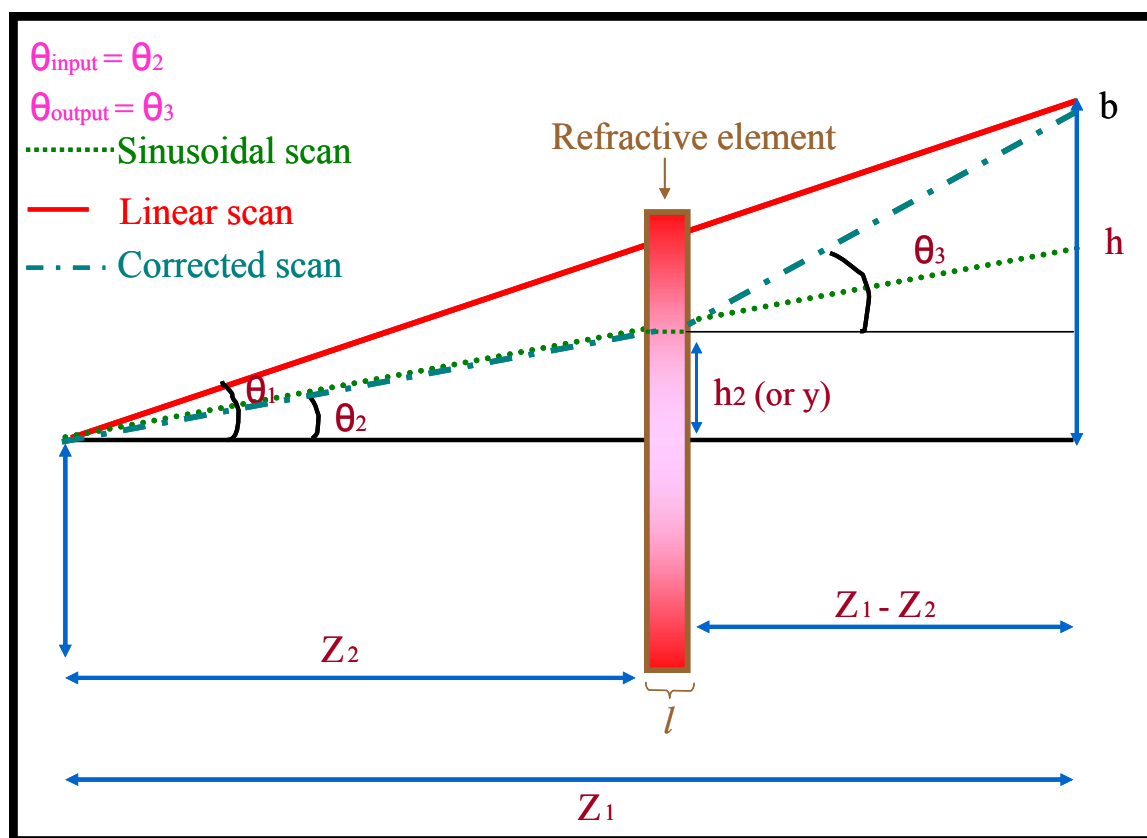
Fig. 7. Diffractive element for the fourth order solution with (a) $\frac{y_0}{\Delta D} = 100$, $y_0=1\text{cm}$ and

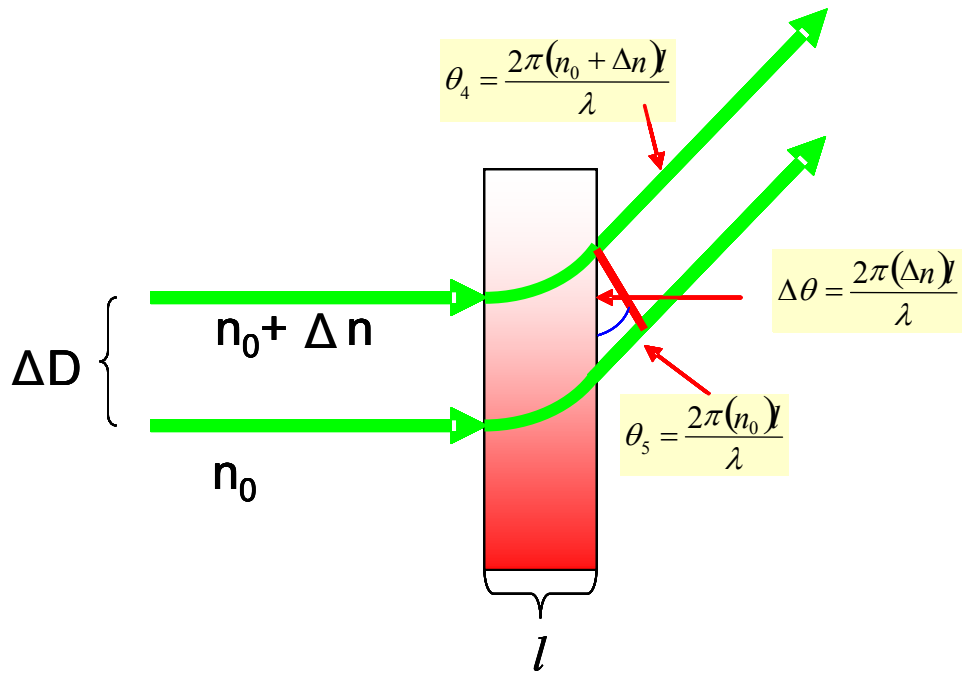
$\Delta D=0.01\text{ cm}$, b) $\frac{y_0}{\Delta D} = 200$, $y_0=1\text{cm}$ $\Delta D=0.005\text{ cm}$, (c) $\frac{y_0}{\Delta D} = 500$, $y_0=1\text{cm}$ $\Delta D=0.002$

cm, as generated in MATLAB. The calculated grating spacing is a smooth chirp. The

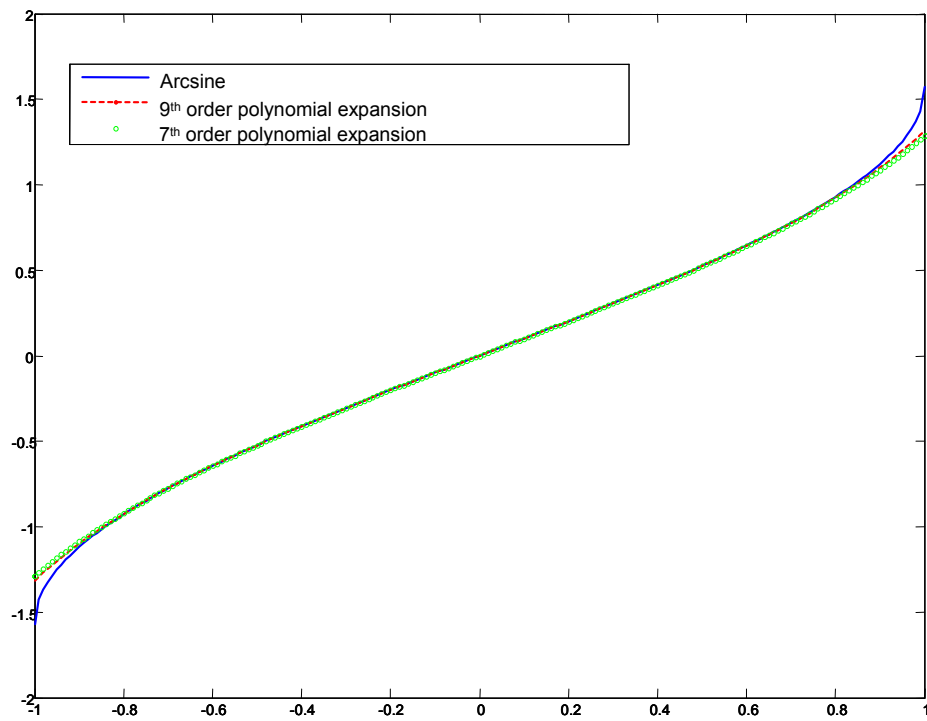
slight banding seen in the figures is an artifact of the way MATLAB displays these patterns.

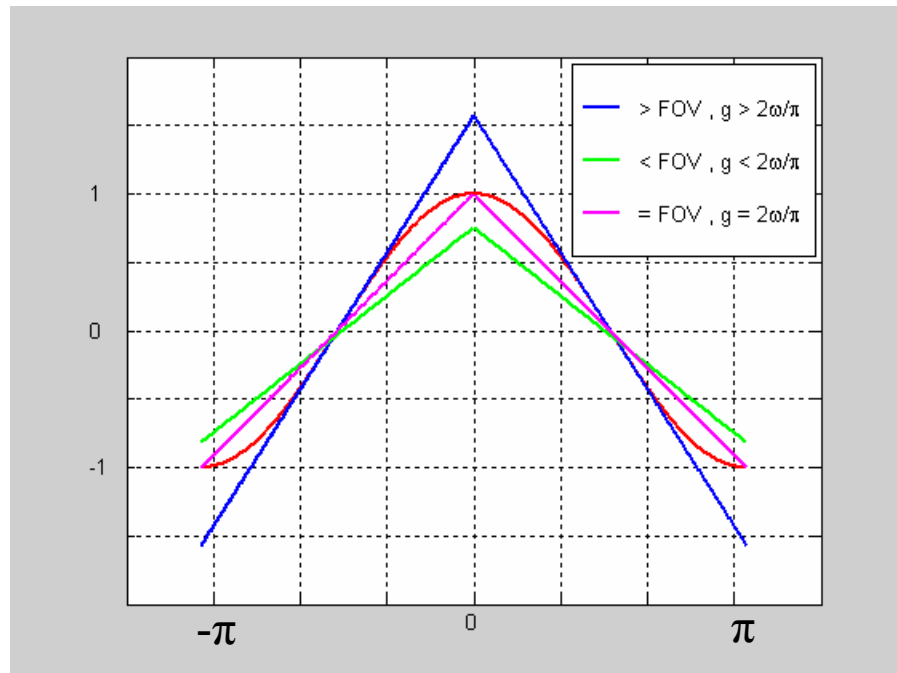
Fig. 8. Plots of the graded index of refraction as a function of the normalized distance from the optical axis for the fourth order solutions (a) For constant y_0 and varying beam diameters ΔD (b) For varying y_0 and constant beam diameter ΔD .

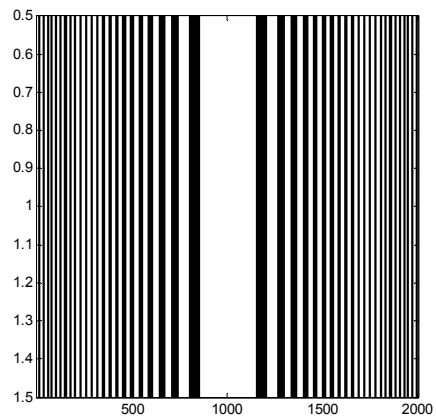




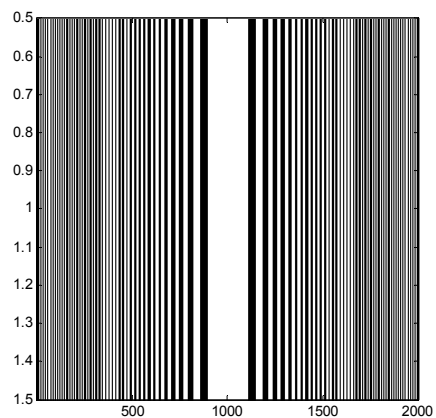
Graded index of
refraction medium



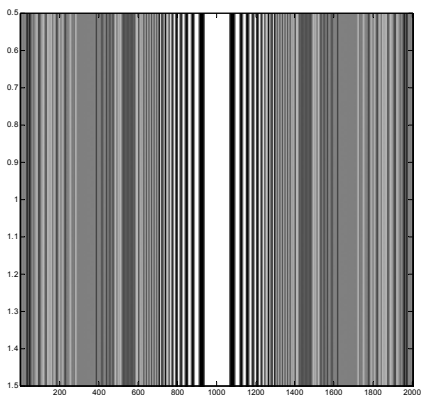




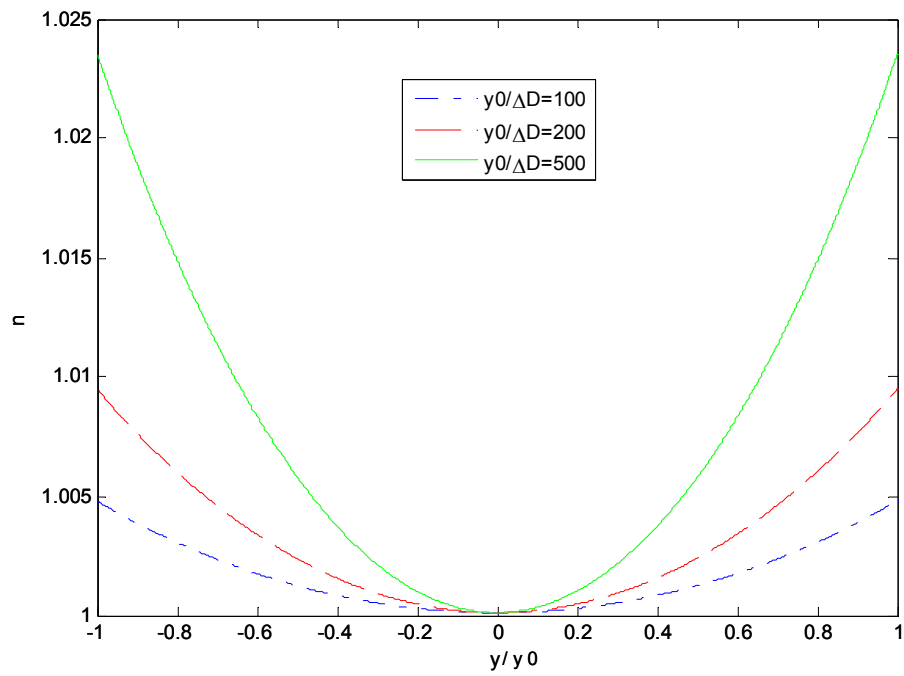
(a)



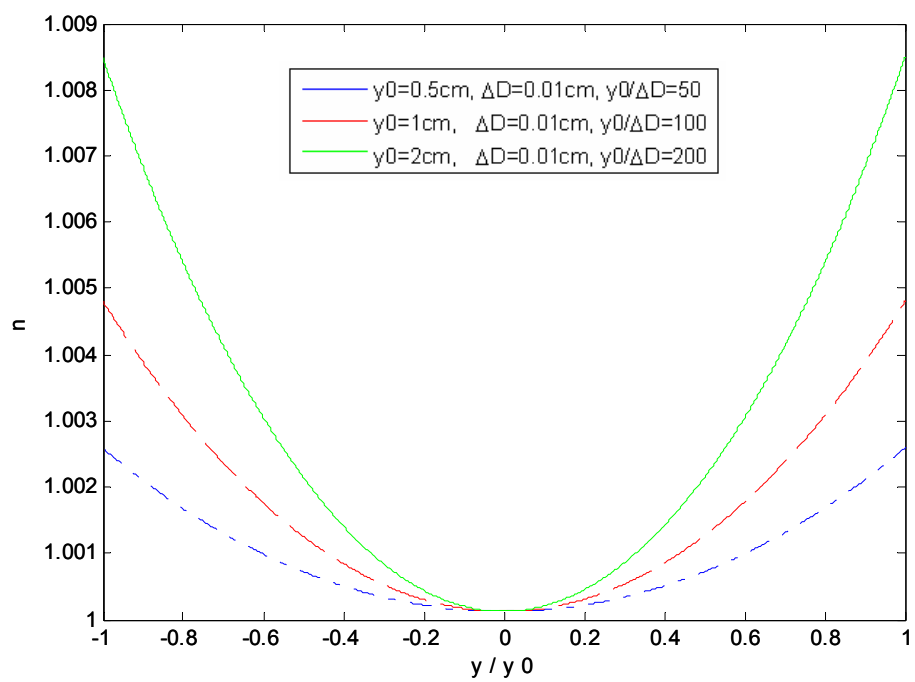
(b)



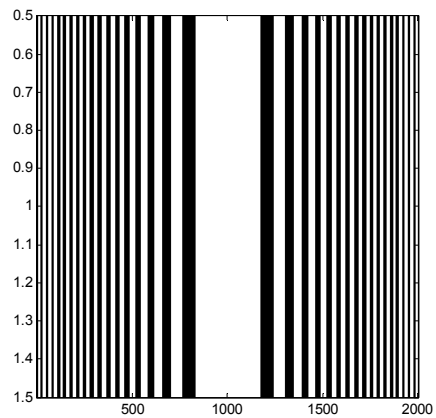
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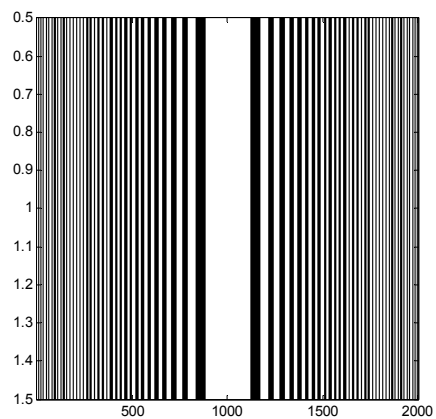
(a)



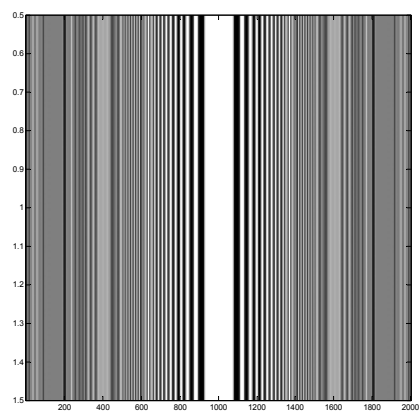
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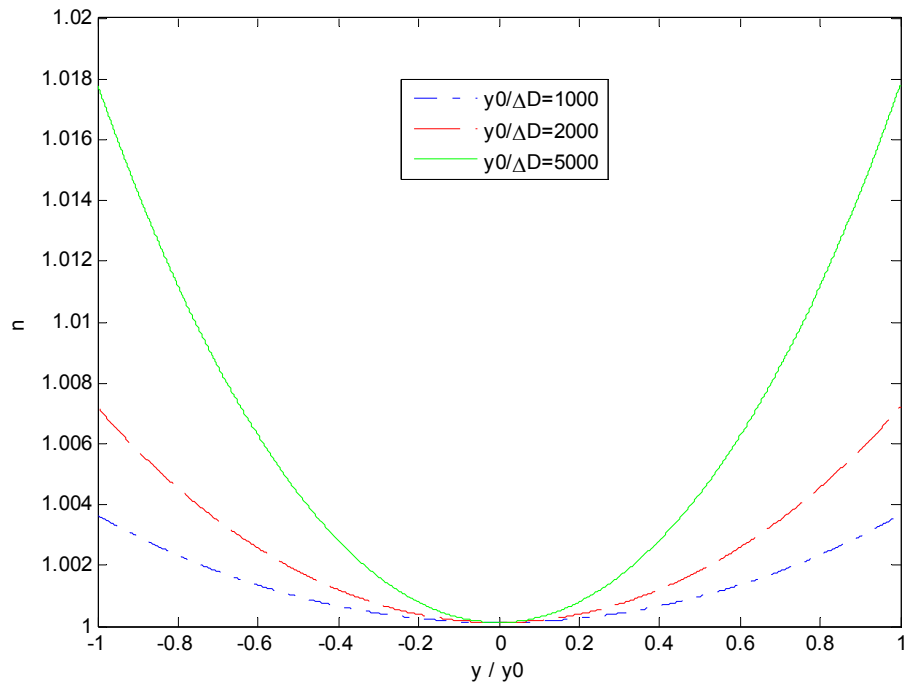
(a)



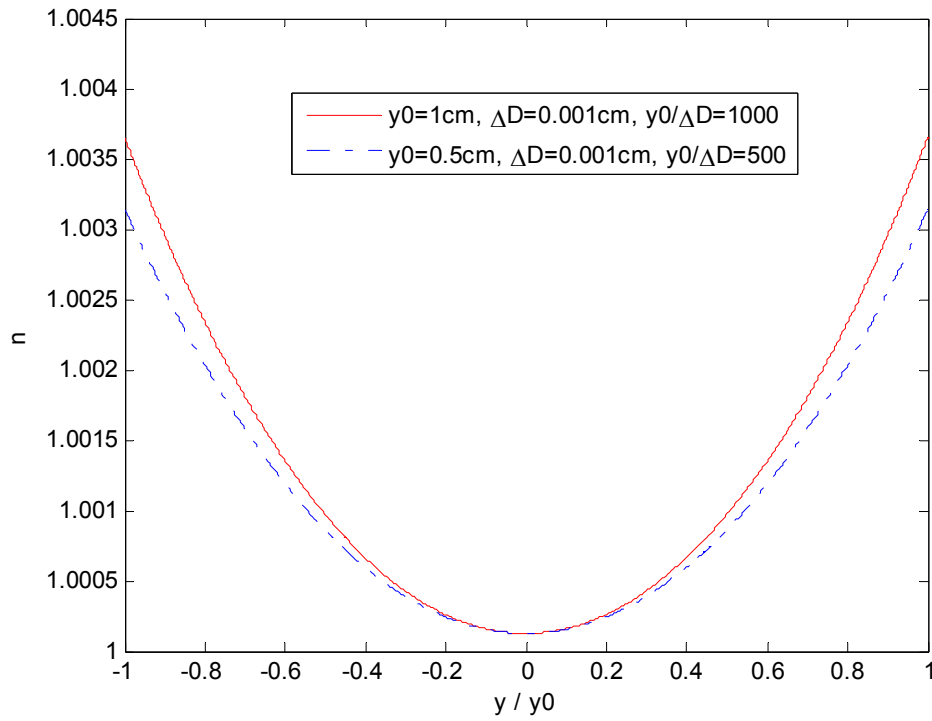
(b)



(c)



(a)



(b)